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DESIGN AND ANALYSIS OF 90-DEGREE PHASE DIFFERENCE NETWORKS

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1. INTRODUCTION

The "Dome filter" was originally described by R. B. Dome[1] and consists of a pair of second-degree all-pass networks with a phase shift difference in the vicinity of 90 degrees over a wide bandwidth. Dome's article shows three different implementations - one LC and two RC circuits - and gives equations for the component values based on an apparently arbitrary (within limits) parameter (s). He suggests frequency parameters for the two all-pass sections which result in a one-sided phase difference curve as shown in figure 1.

Antony published a report[2] on Dome filter design with a computer program for component dimensioning. He points out that there is a relationship between the bandwidth and the ripple of the phase error, which is determined by Dome's parameter s. He suggests to shift the phase difference response down, as shown in figure 2, to obtain equal positive and negative deviations from the nominal value. For a given bandwidth, this design modification almost halves the error or, conversely for a given maximum error, it almost doubles the bandwidth.

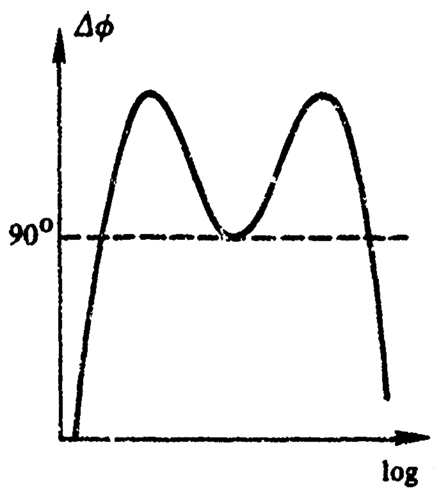

Figure 1. Phase response versus frequency obtained with Dome's dimensioning.

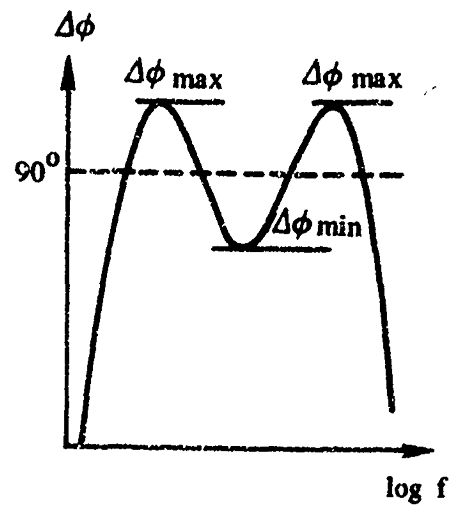

Figure 2. Equal-ripple phase response.

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[1] R. B. Dome, "Wideband Phase Shift Networks," Electronics, Dec 1946, p 112 vol. 4.
[2] R. T. Antony, "Dome Filter Design and Analysis Program," HDL TM-70-26, 1970.

Another RC realization of the Dome filter with a differential amplifier was given by Sebol.[3] He uses as filter parameters tabulated data published by Bedrosian,[4] which also permit the design of higher order phase difference networks by cascading second-degree all-pass sections. The phase response corresponds to figure 2.

2. OBJECTIVE OF THIS REPORT

Neither one of the cited papers presents much of the theoretical background of the Dome filter, the knowledge and understanding of which can add significantly to the designer's confidence. This is one gap which this report intends to fill. While the higher-order filters are not accessible by elementary analysis, the second order (Dome) filter is readily so.

The cited papers present various graphs showing the relationship between the filter parameters, bandwidth and ripple; Antony's computer program calculates the component values from the phase deviation.* This report derives the calculation of the filter parameters based, at the designer's choice, either on the required bandwidth or on the permissible phase error. It also shows the flexibility which exists in dimensioning component values.

Dome uses an arbitrary capacitance ratio in his formulas for the component values, but deprives the designer of one degree of freedom. Antony uses this capacitance ratio in his computer program. Also, the user of his program seems to have little choice as to the component determining the impedance level of the filter. However, as this paper shows, any component of a filter section may be selected for this purpose, and the impedance of the two sections need not necessarily be the same. It is deemed important to acquaint the designer with the flexibility available in this circuit so that he may realize it with as low an impedance spread as possible and as many as possible standard values for the capacitors, because non-standard capacitors are expensive and hard to get.

The calculation of the component values for Sebol's circuit is also summarized to make it available to the user in the same frame of reference.

As this paper shows, the calculation method for filter parameters and components, even with the described full flexibility, is not unduly complicated, requiring only a desk calculator.

Finally, this report presents worst-case analyses of the effects of component tolerances on the phase difference for a Dome circuit and the Sebol circuit. Computer programs for this purpose, and representative sample outputs are listed.

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[3]Sebol, R., "Design of Active 90-Degree Phase Difference Networks," HDL-TM-68-18

[4]Bedrosian, S.D., "Normalized Design of 90-Degree Phase Difference Networks," IRE Trans. on Circuit Theory, June 1960.

*With some error: in one particular case the component values from the computer program were up to three percent off.

3. DERIVATION OF THE FILTER PARAMETERS

3.1 Frequency Response of the Phase Angle

The transfer function of a second-degree all-pass network with the complex frequency p is (fig. 3)[5]

$$\frac{V_2(p)}{V_1} = \frac{(p + p_1)\,(p + \overline{p}_1)}{(p - p_1)\,(p - \overline{p}_1)} \tag{1}$$

Where $p_1 = -\alpha + j\beta$.

It has two conjugate-complex poles and two conjugate-complex zeros located symmetrically to the poles with respect to both the origin and $j\omega$-axis. The frequency response can be written as

$$\frac{V_2(\omega)}{V_1} = \frac{\left[-\alpha + j\,(\omega+\beta)\right]\left[-\alpha + j\,(\omega-\beta)\right]}{\left[\alpha + j\,(\omega-\beta)\right]\left[\alpha + j\,(\omega+\beta)\right]} \tag{2}$$

$$= \text{Magn.}\,\frac{\angle\phi_1 \cdot \angle\phi_2}{\angle\phi_3 \cdot \angle\phi_4} = \text{Magn.}\,\angle\phi$$

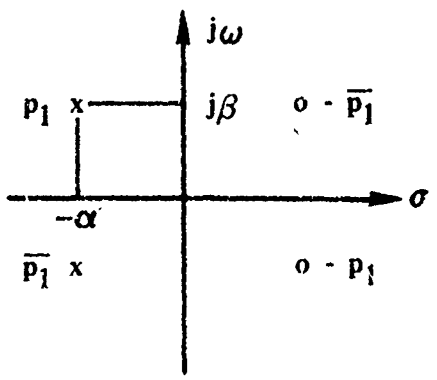

Figure 3. Poles and zeros of second-degree all-pass.

[5] see, for instance, L. Weinberg, "Network Analysis and Synthesis," McGraw Hill, 1962, p 285.

with the phase angle $\phi = \phi_1 + \phi_2 - \phi_3 - \phi_4$.

The phase angles of the individual terms are expressed by

$$\tan\phi_3 = \frac{\omega-\beta}{\alpha}$$

$$\tan\phi_4 = \frac{\omega+\beta}{\alpha}$$

$$\tan\phi_1 = \frac{\omega+\beta}{-\alpha} = \tan\left(\pi-\phi_4\right)$$

$$\tan\phi_2 = \frac{\omega-\beta}{-\alpha} = \tan\left(\pi-\phi_3\right)$$

thus

$$\phi = -2\left(\phi_3+\phi_4\right)$$

$$\tan\left(\phi_3+\phi_4\right) = \frac{2\omega\alpha}{\alpha^2+\beta^2-\omega^2}$$

$$\phi = -2\arctan\frac{2\omega\alpha}{\alpha^2+\beta^2-\omega^2} \tag{3}$$

$$= \arctan\frac{4\omega\alpha\left(\omega^2-\alpha^2-\beta^2\right)}{\left(\omega^2-\alpha^2-\beta^2\right)^2-4\omega^2\alpha^2} \tag{4}$$

and substituting $\omega = 2\pi f$

$$\phi = \arctan\frac{\frac{2f\alpha}{\pi}\left(f^2-\frac{\alpha^2+\beta^2}{4\pi^2}\right)}{\left(f^2-\frac{\alpha^2+\beta^2}{4\pi^2}\right)^2-\frac{f^2\alpha^2}{\pi^2}}.$$

Comparing this result with Dome's equation

$$\tan \phi = \frac{2s\, f\, fo \left(f^2 - fo^2\right)}{\left(f^2 - fo^2\right)^2 - \left(s\, fo\, f\right)^2} \tag{5}$$

shows that Dome's parameters s, f_o are related to α, β by

$$fo = \frac{\sqrt{\alpha^2 + \beta^2}}{2\pi}$$

$$s = \frac{\alpha}{\pi fo} = \frac{2\alpha}{\sqrt{\alpha^2 + \beta^2}} . \tag{6}$$

Using (6) to eliminate α and β from (3) yields

$$\phi = 2 \arctan \frac{f\, fo}{f^2 - fo^2}\, s$$

where f and f_o can be substituted by the frequency function

$$v = \frac{f}{fo} - \frac{fo}{f} \tag{7}$$

resulting in

$$\phi = 2 \arctan \frac{s}{v} . \tag{8}$$

The function v is often used to describe the response of tuned circuits or band pass filters which are symmetrical (although not linearly) about a frequency f_o, corresponding to v = 0. So the phase response of this all-pass network is in the same way symmetrical about f_o, which could be called its center frequency; $\phi\,(f_o) = -180^o$.

The dome filter uses two all-pass sections connected as in figure 4. Of interest is the phase difference of the output signals:

$$\Delta\phi = \phi_2 - \phi_1$$

$$\frac{\Delta\phi}{2} = \arctan \frac{\dfrac{s_2}{v_2} - \dfrac{s_1}{v_1}}{1 + \dfrac{s_1 s_2}{v_1 v_2}} \tag{9}$$

where v_1 and v_2 are defined by (7) with the center frequencies f_{o1} and f_{o2}, respectively. An example of the phase shift with some arbitrary parameters is shown in figure 5.

3.2 Condition For Equal Maxima

Setting the derivative $\frac{d\Delta\phi}{df}$ equal to zero, solving for the locations of the extrema f_{max1}, f_{max2}, f_{min} and letting $\Delta\phi_{max1} = \Delta\phi_{max2}$ would result in a condition between the filter parameters for equal maxima. However, this is too unwieldy to be done in a general way. But it turns out that

$$s_1 = s_2 = s \tag{10}$$

is one condition resulting in equal maxima, as will be shown. For this purpose the following new frequency function is introduced (fig. 6):

$$z = \frac{f}{fm} + \frac{fm}{f} \tag{11}$$

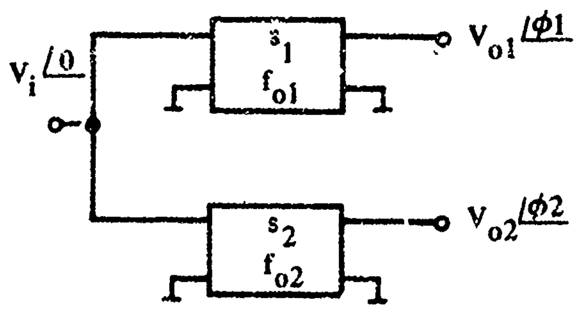

Figure 4. Base dome filter.

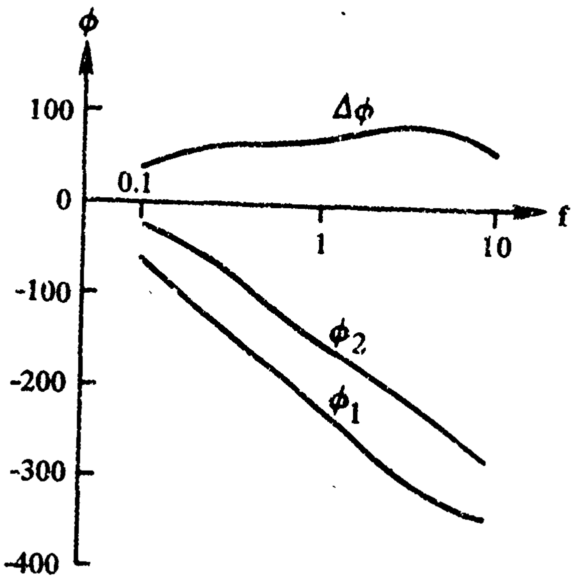

Figure 5. Phase shift and phase difference of an arbitrary dome filter ($f_{o1} = 0.6$; $s_1 = 3.5$; $f_{o2} = 2.0$; $s_2 = 4.6$)

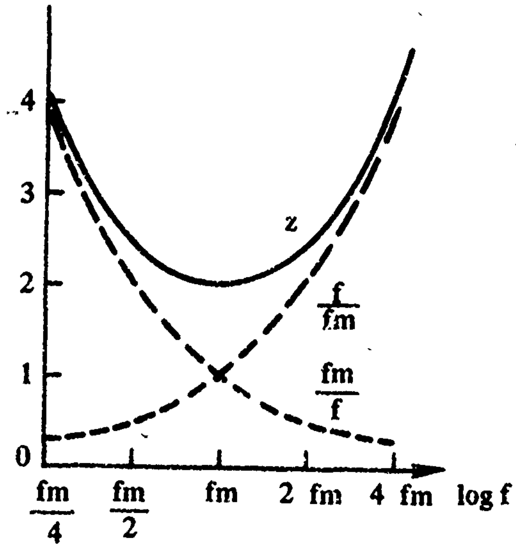

Figure 6. Frequency function z.

where

$$fm = \sqrt{f_{o1} f_{o2}} \quad . \tag{12}$$

With

$$b = \sqrt{\frac{f_{o2}}{f_{o1}}} \tag{13}$$

the center frequencies of the two all-pass sections can be expressed as

$$f_{o1} = \frac{fm}{b}$$

$$f_{o2} = b\,fm \quad . \tag{14}$$

Using these frequency functions and equation 10, equation 9 can be written as a function of z:

$$\frac{\Delta\phi}{2} = \arctan \frac{sz\left(b - \frac{1}{b}\right)}{z^2 - \left(b + \frac{1}{b}\right)^2 + s^2} \ . \tag{15}$$

Under the condition of (10), the phase difference $\Delta\phi$ is obviously symmetrical about f_m, since there are always two values of f (geometrically symmetrical to f_m) which give the same z, and $\Delta\phi$ is a function of z only. So the z-value z_{max} where (15) has a maximum translates into two frequencies, located on opposite sides of f_m, where $\Delta\phi$ has the same maximum amplitude $\Delta\phi(z_{max})$.

z_{max} could be found by setting $\frac{d\Delta\phi}{dz} = 0$, but this procedure does not also yield the minimum. The location of the minimum can be ascertained by re-introducing the frequency into (15) and differentiating with respect to f. As it turns out,

$$f_{min} = f_m$$

corresponding to $z_{min} = 2$; at this point, $\Delta\phi(z)$ has an absolute, not a relative minimum. The frequencies of the maxima are

$$\frac{f_{max}}{f_m} = \frac{\sqrt{s^2 - \left(b + \frac{1}{b}\right)^2} \pm \sqrt{s^2 - \left(b + \frac{1}{b}\right)^2 - 4}}{2} \ . \tag{16}$$

The values of the function at the minimum and at the maxima are as follows:

$$\frac{\Delta\phi_{min}}{2} = \arctan \frac{2s\left(b - \frac{1}{b}\right)}{s^2 - \left(b - \frac{1}{b}\right)^2} \tag{17}$$

$$\frac{\Delta\phi_{max}}{2} = \arctan \frac{s\left(b - \frac{1}{b}\right)}{2\sqrt{s^2 - \left(b - \frac{1}{b}\right)^2 - 4}} \ . \tag{18}$$

3.3 Conditions for Equal Ripple Around 90 Degrees

For most practical applications, a phase difference of 90 degrees is required. The condition for equal positive and negative maximum errors with respect to 90 degrees (fig. 7) is

$$\frac{\Delta\phi\max + \Delta\phi\min}{2} = 90^{\circ}$$

$$\tan\left(\frac{\Delta\phi\max}{2} + \frac{\Delta\phi\min}{2}\right) = \frac{\tan\frac{\Delta\phi\max}{2} + \tan\frac{\Delta\phi\min}{2}}{1 - \tan\frac{\Delta\phi\max}{2}\tan\frac{\Delta\phi\min}{2}} = \tan 90^{\circ} = \infty$$

$$1 - \tan\frac{\Delta\phi\max}{2}\tan\frac{\Delta\phi\min}{2} = 0$$

which, with (17) and (18), leads to

$$\frac{s^2\left(b - \frac{1}{b}\right)^2}{\sqrt{s^2 - \left(b - \frac{1}{b}\right)^2 - 4\left[s^2 - \left(b - \frac{1}{b}\right)^2\right]}} = 1. \qquad (19)$$

This equation may be combined with one other condition to solve for the parameters s and b: either the maximum deviation or the required bandwidth.

3.4 Bandwidth

The edge frequencies (fig. 7) that describe the useful frequency range are defined as the frequencies where the error of the phase difference is the same as at the minimum, δ, which is equal in magnitude to the deviation at the maximum. At this band edge, the frequency function z has the value

$$z_L = \frac{f_1}{fm} + \frac{fm}{f_1} = \frac{f_2}{fm} + \frac{fm}{f_2} ;$$

solved for f_m and z_L:

$$f_m = \sqrt{f_1 f_2} \tag{20}$$

$$z_L = \sqrt{\frac{f_1}{f_2}} + \sqrt{\frac{f_2}{f_1}} \tag{21}$$

or solved for f_1 and f_2

$$f_2 = \left(\frac{z_L}{2} + \sqrt{\frac{z_L^2}{4} - 1}\right) f_m$$

$$f_1 = \frac{f_m^2}{f_2} \,. \tag{22}$$

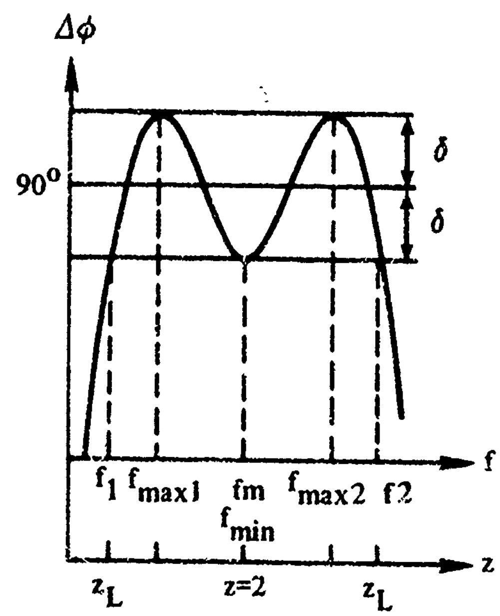

Figure 7. Definition of edge frequencies f_1, f_2.

3.5 Filter Parameters for a Specified Bandwidth

If the edge frequencies f_1, f_2 are specified, f_m and z_L follow from (20) and (21). At the band edge,

$$\Delta\phi(z_L) = \Delta\phi\,\mathrm{min}.$$

The two sides of this equation are given by (15) and (17). Combined with the condition of equal ripple (19), a solution for the parameters s and b is obtained:

$$s^2 = \frac{z_L^2-4}{z_L-2} \pm \sqrt{\left(\frac{z_L^2-4}{z_L-2}\right)^2 + 2\,\frac{z_L^2-4}{z_L-2}\,\sqrt{2\,\frac{z_L^2-4}{z_L-2} - 4}} \qquad \text{(23) (pos. solution)}$$

$$b^2 = 1 + \frac{s^2}{2} - \frac{z_L^2-4}{z_L-2} \overset{+}{(-)} \sqrt{\left(1 + \frac{s^2}{2} - \frac{z_L^2-4}{z_L-2}\right)^2 - 1} \qquad (24)$$

$\left(+ \text{ sign if } f_{o2} > f_m > f_{o1}\right)$.

The phase deviation δ occuring in this case follows from

$$90^\circ - \delta = \Delta\phi\,\mathrm{min}$$

$$\tan\left(45^\circ - \frac{\delta}{2}\right) = \frac{1 - \tan\frac{\delta}{2}}{1 + \tan\frac{\delta}{2}} = \tan\frac{\Delta\phi\,\mathrm{min}}{2} \;;$$

using (17) and solving for $\tan\frac{\delta}{2}$:

$$\tan\frac{\delta}{2} = \frac{s^2 - \left(b - \frac{1}{b}\right)^2 - 2s\left(b - \frac{1}{b}\right)}{s^2 - \left(b - \frac{1}{b}\right)^2 + 2s\left(b - \frac{1}{b}\right)} \,. \qquad (25)$$

The order of calculation is shown in figure 8.

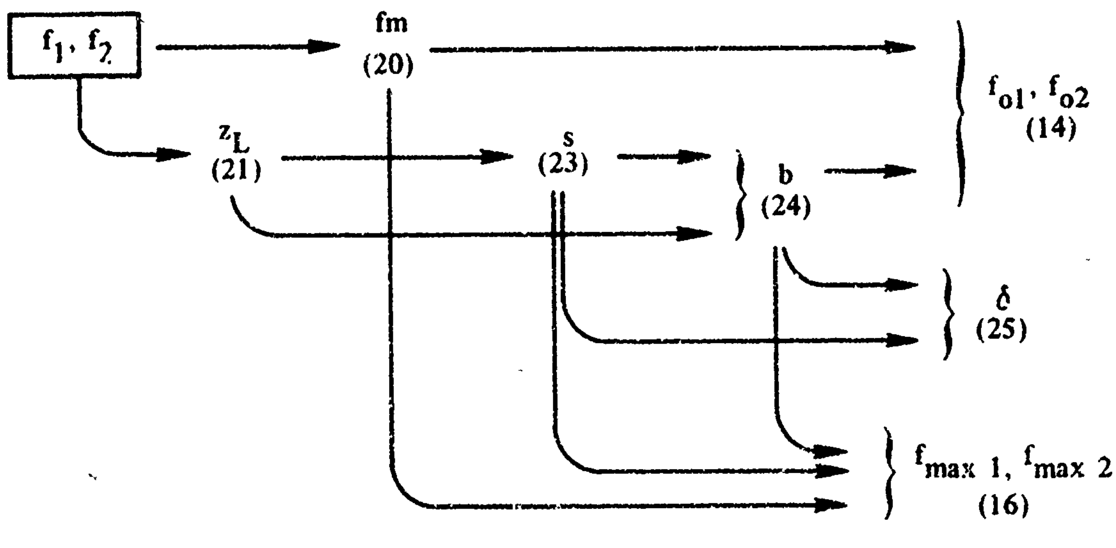

Figure 8. Order of calculation, and equations used, when the band edges are specified.

3.6 Filter Parameters For a Specified Deviation

Instead of the bandwidth, the maximum deviation δ from 90 degrees may be specified. The two equations to be solved for s and b are (19) and (25). With the abbreviation

$$D = \frac{1 - \tan\frac{\delta}{2}}{1 + \tan\frac{\delta}{2}}, \tag{26}$$

b is eliminated by

$$b = \pm\sqrt{\frac{s^2-4}{\sqrt{s^2D^2+4}} + \sqrt{\frac{s^2-4}{s^2D^2+4}} + 1} \tag{27}$$

and a polynom of 8th degree in s is obtained:

$$F(s) \equiv \left(s^4D^2+16\right)^2 - 16\,s^2\left(s^2-4\right)\left(s^2D^2+4\right) = 0. \tag{28}$$

It may be solved numerically using Newton's method:

$$s_2 = s_1 - \frac{F(s_1)}{F'(s_1)} ;$$

the derivative is

$$F'(s) = 8D^6 s^7 - 96D^2 s^5 + 128 s^3 \left(D^4 + 2D^2 - 2\right) + 512 s .$$

After the solution of s has been found, b is calculated from (27). The resulting bandwidth is obtained again by considering that, at the band edge $z = z_L$,

$$\Delta\phi\left(z_L\right) = \Delta\phi \min .$$

Substituting both parts of this equation by (15) and (17) and solving for z_L:

$$z_L = \frac{1}{4}\left[s^2 - \left(b - \frac{1}{b}\right)^2\right] + \sqrt{\frac{1}{16}\left[s^2 - \left(b - \frac{1}{b}\right)^2\right]^2 - \left[s^2 - \left(b - \frac{1}{b}\right)^2\right] + 4} \tag{29}$$

The edge frequencies are obtained from (22). Figure 9 summarizes the order of calculations. The following tabulation shows some numerical examples:

δ	2 deg	3 deg	4 deg
s	3.868144	4.102830	4.325051
b	2.050514	2.110651	2.165476
z_L	4.260048	5.076946	5.901764
f_2/f_1	16.0358	23.7332	32.8003

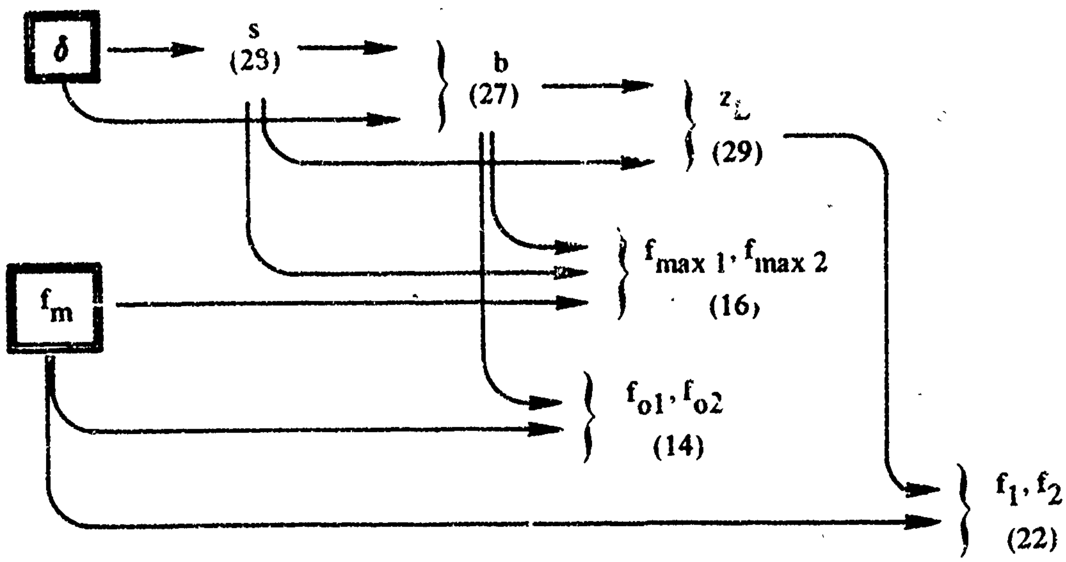

Figure 9. Order of calculation, and equations used, when the maximum deviation is specified.

For $\delta = 0$, the smallest possible value of s is obtained (s = 3.107543). Going back to the original parameters α, β of the all-pass network and solving (6) for β:

$$\beta = \pi f_o \sqrt{4 - s^2}$$

shows that, for all practical filters, β is imaginary, $j\beta$ is real and all poles and zeros are located on the real axis (fig. 10). Hence, the network is suited for RC realization.

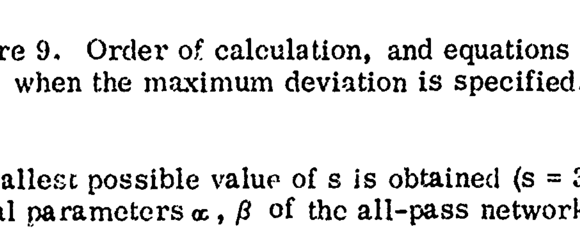

Figure 10. Pole and zero location for s > 2.

4. RC REALIZATION WITH BALANCED GENERATOR

One of the implementations of the all-pass filter suggested by Dome is an RC network fed by two out-of-phase signals, *figure* 11. In view of the worst-case analysis performed later, a possible difference of the magnitudes of the input voltages has been considered. The frequency response of the transfer function is:

$$\frac{\vec{V}_3}{V_1} = \frac{-(1+\Delta V)\, C_2}{C_2 + C_3}$$

$$\cdot \frac{(j\omega)^2 - j\omega\left[\frac{1}{(1+\Delta V)\, R_1 C_2} - \frac{1}{R_1 C_1} - \frac{1}{R_2 C_2}\right] + \frac{1}{R_1 R_2 C_1 C_2}}{(j\omega)^2 + j\omega\left[\frac{\frac{1}{R_2} + \frac{1}{R_3}}{C_2 + C_3} + \frac{1}{R_1 C_1} + \frac{1}{R_1 (C_2 + C_3)}\right] + \frac{\frac{1}{R_2} + \frac{1}{R_3}}{R_1 C_1 (C_2 + C_3)}} \tag{30}$$

4.1 All-pass Conditions

The all-pass conditions of the circuit of figure 11 are derived for $\Delta V = 0$. The general all-pass transfer function (2) may be written as

$$\frac{V_2}{V_1} = \frac{(j\omega)^2 - 2\alpha j\omega + \alpha^2 + \beta^2}{(j\omega)^2 + 2\alpha j\omega + \alpha^2 + \beta^2} \; . \tag{31}$$

Compared to (30), it follows that (31) describes an all-pass if

$$\frac{\frac{1}{R_2} + \frac{1}{R_3}}{C_2 + C_3} + \frac{1}{R_1 C_1} + \frac{1}{R_1 (C_2 + C_3)} = \frac{1}{R_1 C_2} - \frac{1}{R_1 C_1} - \frac{1}{R_2 C_2}$$

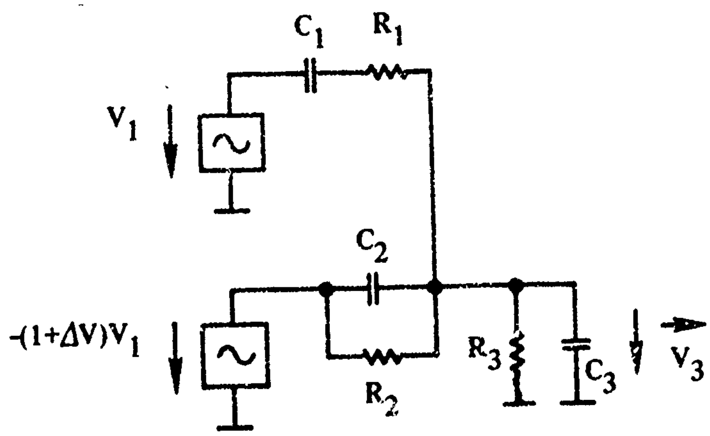

Figure 11. All-pass circuit with balanced generator.

and

$$\frac{\frac{1}{R_2}+\frac{1}{R_3}}{R_1 C_1 \left(C_2+C_3\right)} = \frac{1}{R_1 R_2 C_1 C_2}$$

These two equations may be simplified to

$$\frac{C_2}{C_3} = \frac{R_3}{R_2} \tag{32}$$

$$\frac{C_2}{C_1} + \frac{R_1}{R_2} = \frac{1/2}{1+\frac{R_3}{R_2}} \, . \tag{33}$$

With the abbreviations

$$c = \frac{C_1}{C_2}$$

$$r = \frac{R_1}{R_2}$$

and recalling from (6) that

$$\alpha^2 + \beta^2 = {\omega_o}^2 = \frac{1}{R_1 R_2 C_1 C_2}$$

equation (30) can be written if the all-pass conditions are met as

$$\frac{\vec{V}_3}{V_i} = \left[-1+2\left(\frac{1}{c}+r\right)\right] \frac{\left(\frac{j\omega}{\omega_o}\right)^2 - \frac{j\omega}{\omega_o}\left(\sqrt{\frac{c}{r}} - \frac{1}{\sqrt{cr}} - \sqrt{cr}\right) + 1}{\left(\frac{j\omega}{\omega_o}\right)^2 + \frac{j\omega}{\omega_o}\left(\sqrt{\frac{c}{r}} - \frac{1}{\sqrt{cr}} - \sqrt{cr}\right) + 1} \tag{34}$$

4.2 Dimensioning

As shown by (34) the transfer function of the all-pass circuit is expressed by the angular center frequency ω_0 and the two circuit parameters c and r. The desired filter parameter s (par. 3.5 and 3.6) provides one condition between then, so one parameter is selectable. Recalling from (6) that

$$s = \frac{2\alpha}{\omega_o}$$

and comparing (34) with (31) shows that

$$s = \sqrt{\frac{c}{r}} - \frac{1}{\sqrt{cr}} - \sqrt{cr} \; .$$

As it is convenient to select the capacitance ratio, this equation is solved for the resistance ratio

$$\sqrt{r} = \frac{-s}{2\sqrt{c}} + \sqrt{\frac{s^2}{4c} + 1 - \frac{1}{c}} \; . \tag{35}$$

The dimensioning continues by calculating the ratios R_3/R_2 and C_2/C_3, which are equal according to (32) and follow from c, r using (33):

$$\frac{C_2}{C_3} = \frac{R_3}{R_2} = a = \frac{1}{2\left(\frac{1}{c}+r\right)} - 1 \; . \tag{36}$$

The ratio c has to be selected so that both r and a are positive real numbers. Figure 12 shows as an example the range of possible values for the filter parameter s = 4.

Dome and Antony arbitrarily set

$$R_1C_1 = R_2C_2 \; .$$

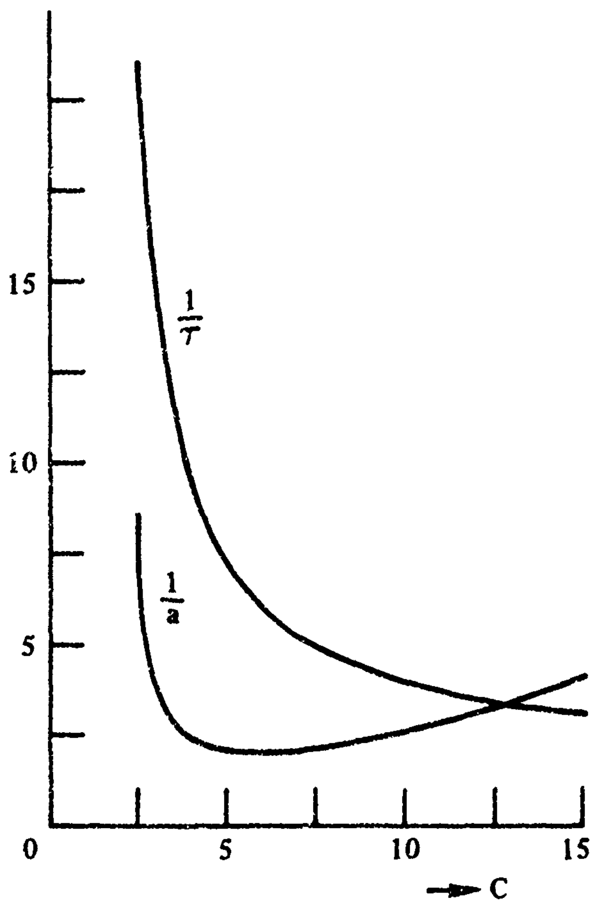

Figure 12. Possible circuit parameters for s = 4.

which means

$$r = \frac{1}{c}$$

and

$$c = s + 2$$

$$a = \frac{c}{4} - 1 \,.$$

After a set of circuit parameters c, r, a has been found, continue by selecting one of the three

resistors	or	capacitors
$R_1 \overset{r}{\longleftrightarrow} R_2 \overset{a}{\longleftrightarrow} R_3$		$C_1 \overset{c}{\longleftrightarrow} C_2 \overset{a}{\longleftrightarrow} C_3$

then obtain

$C_1 = \dfrac{\sqrt{rc}}{\omega_o R_1}$	$R_1 = \dfrac{\sqrt{rc}}{\omega_o C_1}$	(37)
$C_1 \overset{c}{\longleftrightarrow} C_2 \overset{a}{\longleftrightarrow} C_3$	$R_1 \overset{r}{\longleftrightarrow} R_2 \overset{a}{\longleftrightarrow} R_3$	

4.3 Effect of Component Tolerances

With

nominal parameters ω_o, a, c, r;

nominal components R_{1o}, R_{2o}, R_{3o}, C_{1o}, C_{2o}, C_{3o}, $\Delta V = 0$;

actual components R_1, R_2, R_3, C_1, C_2, C_3, ΔV;

the transfer function of one all-pass section (31) can be written as:

$$\frac{\vec{V}_3}{V_1} = \frac{-(1+\Delta V)\,C_2}{C_2 + C_3} \cdot \frac{\left(\frac{j\omega}{\omega_o}\right)^2 - \frac{j\omega}{\omega_o} E_1 + E_2}{\left(\frac{j\omega}{\omega_o}\right)^2 + \frac{j\omega}{\omega_o} E_3 + E_4} \tag{38}$$

where

$$E_1 = \frac{R_{10}}{R_1}\frac{C_{20}}{C_2}\sqrt{\frac{c}{r}} - \frac{R_{10}}{R_1}\frac{C_{10}}{C_1}\frac{1}{\sqrt{cr}} - \frac{R_{20}}{R_2}\frac{C_{20}}{C_2}\sqrt{cr}$$

$$E_2 = \frac{R_{10}}{R_1}\frac{R_{20}}{R_2}\frac{C_{10}}{C_1}\frac{C_{20}}{C_2}$$

$$E_3 = \sqrt{cr}\,\frac{\frac{R_{20}}{R_2} + \frac{1}{a}\frac{R_{30}}{R_3}}{\frac{C_2}{C_{20}} + \frac{1}{a}\frac{C_3}{C_{30}}} + \frac{R_{10}}{R_1}\frac{C_{10}}{C_1}\frac{1}{\sqrt{cr}} + \sqrt{\frac{c}{r}}\,\frac{\frac{R_{10}}{R_1}}{\frac{C_2}{C_{20}} + \frac{1}{a}\frac{C_3}{C_{30}}}$$

$$E_4 = \frac{\frac{R_{10}}{R_1}\left(\frac{R_{20}}{R_2} + \frac{1}{a}\frac{R_{30}}{R_3}\right)}{\frac{C_1}{C_{10}}\left(\frac{C_2}{C_{20}} + \frac{1}{a}\frac{C_3}{C_{30}}\right)}$$

Equation (38) contains only the nominal circuit parameters and ratios of nominal to actual component values. In this way, the effect of percentage variations of components on the frequency response can be studied for a more general network. A computer program, listed in appendix 1, has been written to calculate the worst-case phase difference for combination of specified component tolerances.

Tolerances may be specified for each of the three resistors, for each of the three capacitors and for the symmetry of the input voltages. The program assumes the same tolerances for corresponding elements in the two channels. It further uses the fact that the tolerance of a pair of components (e.g., R_2 in channel 1 and 2) affects the response most if the two components are at opposite ends of their tolerance range (channel 1: $R_2 = R_{20} + \Delta R_2$; channel 2: $R_2 = R_{20} - \Delta R_2$). The program goes through all possible combinations of tolerances (2^7 combinations if all tolerances are specified as non-zero); and in each case sweeps through the frequency range and notes the phase differences at the extrema and the band edges. The smallest and largest values for all combinations at each of these points is printed out.

The band edges are specified as the ratio f_2/f_1, from which the filter parameters s, b, δ are immediately computed (eq. 23, 24, 25). The running frequency variable is $x = \frac{f}{f_m}$, and it is swept from

$$x_1 = \frac{f_1}{f_m} = \sqrt{\frac{f_1}{f_2}} \quad \text{to} \quad x_2 = \frac{f_2}{f_m} = \sqrt{\frac{f_2}{f_1}} .$$

The frequency variable actually substituted into equation (38) is (using (12) and (13)):

For Channel 1	For Channel 2
$\frac{\omega}{\omega_{o1}} = \frac{f}{fm}\, b$	$\frac{\omega}{\omega_{o2}} = \frac{f}{fm} \cdot \frac{1}{b}$.

Since the absolute frequency limits need not be specified, the worst-case analysis is valid for any filter of such configuration characterized by band width f_2/f_1 and implementation parameters c(chan. 1), c(chan. 2).

4.4 Amplifier Considerations

The Dome filter implementation treated in this chapter must be fed by two signals of equal amplitude and opposite phase (fig. 11). A simple one-transistor phase splitter as in figure 13 probably does not suffice when a phase error of a few degrees is desired.* The change of the emitter load $C_2 \| R_2$ with frequency reflects onto the equivalent collector voltage source.

This interaction is considerably reduced in the two-transistor circuit of figure 14. It should work satisfactorily when the emitter load impedances R_1 and $R_E \| R_2 \| C_2$, multiplied by the transistor current gain, each are large compared to the generator impedance Z_G.

---

*supported by experience relayed by K. Sann, HDL

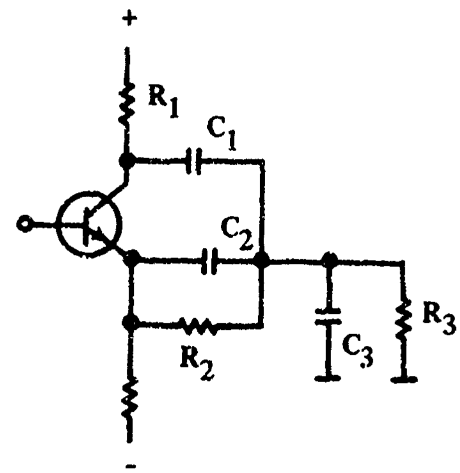

Figure 13. Dome filter fed by simple phase splitter, not recommended.

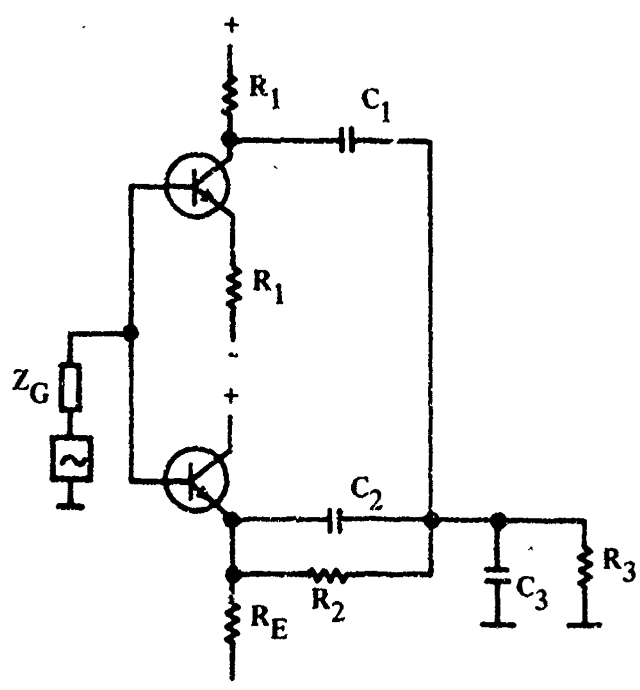

Figure 14. Dome filter fed by two transistors.

Impedance relationships need not be considered when feeding the Dome filter by a pair of integrated-circuit operational amplifiers (fig. 15). One operational amplifier works in the inverting mode, the other in the noninverting mode. With unity gain, as shown, the voltage symmetry depends upon the two resistors R_F being equal. With a resistor tolerance $\frac{\Delta R_F}{R_F}$, the gain tolerance is

$$\Delta V = 2 \frac{\Delta R_F}{R_F} .$$

The amplifiers may provide voltage gain; in that case there are two pairs of resistors whose ratio must be equal.

A phase difference between the outputs of the inverting and noninverting amplifier is possible, particularly at frequencies approaching the upper limit of the operational amplifier; this error is not included in the worst-case analysis.

4.5 Example

Design a phase difference network of 90 degrees with a maximum nominal deviation of ±2 degrees and a bandwidth as great as possible, extending, if possible, from 600 to 10000 Hz.

$$\delta = 2^{\circ}$$

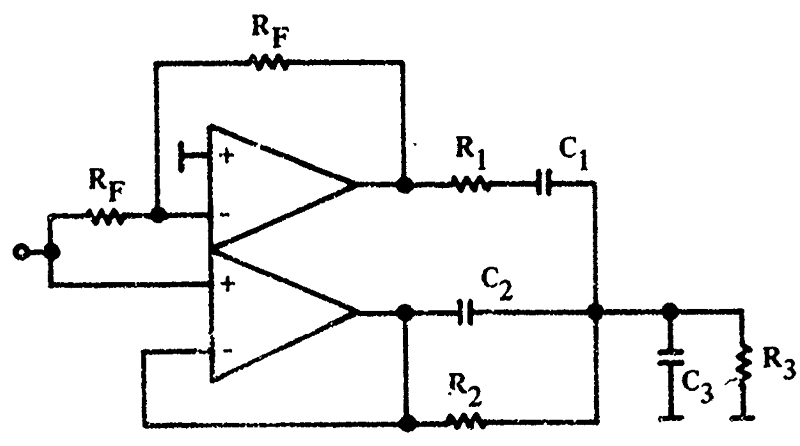

Figure 15. Dome filter fed by two operational amplifiers.

According to the scheme of figure 9:

$$s = 3.868144$$

$$b = 2.050514$$

$$z_L = 4.260048$$

Using the desired edge frequencies to find a suitable center frequency

$$f_m \approx \sqrt{600 \cdot 10000}\ \text{Hz} = 2449.5\ \text{Hz} \longrightarrow$$

$$f_m = 2450\ \text{Hz}.$$

The actual band limits are then

$$f_1 = 610.9\ \text{Hz}$$

$$f_2 = 9826\ \text{Hz}$$

and the other characteristic frequencies are

$$f_{o1} = 1194.82\ \text{Hz}$$

$$f_{o2} = 5023.76\ \text{Hz}$$

$$f_{max1} = 971.3\ \text{Hz}$$

$$f_{max2} \quad 6180\ \text{Hz}\ .$$

It is generally advantageous to select capacitor, rather than resistor values, to be standard values.* For each all-pass section, two capacitors can be selected. In this example, C_1 and C_2 were chosen by trial and error so that C_3 comes close to a standard value too. Depending on the permissible actual phase error, this standard value may be usable. If nothing else, it makes breadboarding easier.

Section 1	Section 2
C_1 = 12 nF	C_1 = 6.8 nF
C_2 = 2.2 nF	C_2 = 1 nF
c = 5.454545	c = 6.800000
r = 0.158105	r = 0.196085
a = 0.464393	a = 0.457115
C_3 = 4.737 nF	C_3 = 2.188 nF
R_1 = 10.31 kΩ	R_1 = 5.380 kΩ
R_2 = 65.20 kΩ	R_2 = 27.44 kΩ
R_3 = 30.28 kΩ	R_3 = 12.54 kΩ

The results of the worst-case analysis are shown in table I. After listing of the parameters, the frequency is swept once with the nominal values to ensure that the input data are correct. The next seven results show the effect of the variation of one component at a time. As it is, R_1 and C_2 affect the phase difference mainly at the higher frequencies, R_2 and C_1 mainly at the lower frequencies. A voltage unbalance shows up, although to a lesser degree, at both ends of the band. The effect of variations of R_3 and C_3 is only about one-fourth of that of the other resistors and capacitors.

With one percent tolerance of all components (except one-half percent for R_F if the circuit shown in figure 15 is used), the possible phase error is almost three degree, in addition to the nominal two degree error with ideal components. If all resistor tolerances are reduced to one-fourth percent, the possible error is close to four degrees, or twice the nominal error. If standard values (±1 percent) are ordered for C_3, its possible variation may be up to 1.7 percent from the theoretical value; as the last output shows, the error is increased only very slightly.

*Although one-percent capacitors may be ordered with any arbitrary nominal value, nonstandard values have higher prices and longer delivery time. This was found to be the case with ceramic capacitors from Aerovox, Erie and Vitramon.

TABLE I. DOME CIRCUIT WITH BALANCED GENERATOR

```
NOMINAL BANDWIDTH F2/F1= 16.0800
NOM. DERIVED PARAM.    S=3.867941    B=2.050461    DELTA=2.00

NOM. REALIZ. CHAN.1    A= 0.46434    C= 5.45450    R= 0.15811
             CHAN.2       0.45706       6.80000       0.19610
**************************************************************
                  F1      1.MAX     MIN     2.MAX     F2
                 ======================================
DP NOM./DEG      88.00    92.00    88.00    92.00    88.00

COMPONENT VARIATION (+/-PCT)  R1     R2     R3     C1     C2     C3     DV
(OPPOSITE IN CHAN.1,2)       -------------------------------------------------
                               1.00 -0.00 -0.00 -0.00 -0.00 -0.00 -0.00

DP MAX /DEG      88.14    92.26    88.64    92.93    88.96
DP MIN /DEG      87.86    91.74    87.35    91.06    87.04

COMPONENT VARIATION (+/-PCT)  R1     R2     R3     C1     C2     C3     DV
(OPPOSITE IN CHAN.1,2)       -------------------------------------------------
                             -0.00  1.00 -0.00 -0.00 -0.00 -0.00 -0.00

DP MAX /DEG      88.71    92.68    88.43    92.11    88.02
DP MIN /DEG      87.29    91.32    87.57    91.89    87.98

COMPONENT VARIATION (+/-PCT)  R1     R2     R3     C1     C2     C3     DV
(OPPOSITE IN CHAN.1,2)       -------------------------------------------------
                             -0.00 -0.00  1.00 -0.00 -0.00 -0.00 -0.00

DP MAX /DEG      88.25    92.25    88.21    92.15    88.12
DP MIN /DEG      87.75    91.74    87.79    91.85    87.88

COMPONENT VARIATION (+/-PCT)  R1     R2     R3     C1     C2     C3     DV
(OPPOSITE IN CHAN.1,2)       -------------------------------------------------
                             -0.00 -0.00 -0.00  1.00 -0.00 -0.00 -0.00

DP MAX /DEG      88.96    92.93    88.65    92.26    88.14
DP MIN /DEG      87.04    91.06    87.35    91.74    87.86

COMPONENT VARIATION (+/-PCT)  R1     R2     R3     C1     C2     C3     DV
(OPPOSITE IN CHAN.1,2)       -------------------------------------------------
                             -0.00 -0.00 -0.00 -0.00  1.00 -0.00 -0.00

DP MAX /DEG      88.02    92.12    88.43    92.67    88.70
DP MIN /DEG      87.98    91.88    87.57    91.32    87.30

COMPONENT VARIATION (+/-PCT)  R1     R2     R3     C1     C2     C3     DV
(OPPOSITE IN CHAN.1,2)       -------------------------------------------------
                             -0.00 -0.00 -0.00 -0.00 -0.00  1.00 -0.00

DP MAX /DEG      88.12    92.14    88.21    92.26    88.26
DP MIN /DEG      87.88    91.85    87.79    91.74    87.74

COMPONENT VARIATION (+/-PCT)  R1     R2     R3     C1     C2     C3     DV
(OPPOSITE IN CHAN.1,2)       -------------------------------------------------
                             -0.00 -0.00 -0.00 -0.00 -0.00 -0.00  1.00

DP MAX /DEG      88.62    92.51    87.99    92.51    88.62
DP MIN /DEG      87.37    91.48    87.99    91.48    87.37
```

TABLE I. DOME CIRCUIT WITH BALANCED GENERATOR (Continued)

COMPONENT VARIATION (+/-PCT) (OPPOSITE IN CHAN.1,2.	R1	R2	R3	C1	C2	C3	DV
	1.00	1.00	1.00	1.00	1.00	1.00	1.00

DP MAX /DEG	90.80	94.89	90.58	94.89	90.80
DP MIN /DEG	85.15	89.07	85.41	89.07	85.15

COMPONENT VARIATION (+/-PCT) (OPPOSITE IN CHAN.1,2)	R1	R2	R3	C1	C2	C3	DV
	0.25	0.25	1.00	1.00	1.00	1.00	0.50

DP MAX /DEG	89.87	93.94	89.78	93.86	89.77
DP MIN /DEG	86.11	90.05	86.22	90.13	86.21

COMPONENT VARIATION (+/-PCT) (OPPOSITE IN CHAN.1,2)	R1	R2	R3	C1	C2	C3	DV
	0.25	0.25	1.00	1.00	1.00	1.70	0.50

DP MAX /DEG	89.95	94.04	89.9[illegible]	94.04	89.95
DP MIN /DEG	86.03	89.95	86.07	89.95	86.03

5. RC REALIZATION WITH DIFFERENTIAL AMPLIFIER

While the circuit discussed in the previous section may be fed by operational amplifiers, the implementation suggested by SEBOL needs an operational amplifier as an active part. It is based on the lattic network of figure 16 with one terminal of the input voltage grounded and the output floating. In the actual circuit of figure 17, this output is connected to a differential (operational) amplifier, and the resistive divider is also utilized as the feedback path. The transfer function is slightly different from the one of figure 16 and the voltage gain is unity.

The frequency response of the circuit of figure 17 is

$$\frac{V_3}{V_1} = \frac{(j\omega)^2 + j\omega\left(\frac{1}{C_1R_1} + \frac{1}{C_2R_2} - \frac{k}{C_1R_2}\right) + \frac{1}{C_1C_2R_1R_2}}{(j\omega)^2 + j\omega\left(\frac{1}{C_1R_1} + \frac{1}{C_2R_2} + \frac{1}{C_1R_2}\right) + \frac{1}{C_1C_2R_1R_2}} \tag{39}$$

where

$$k = \frac{R_4}{R_3}.$$

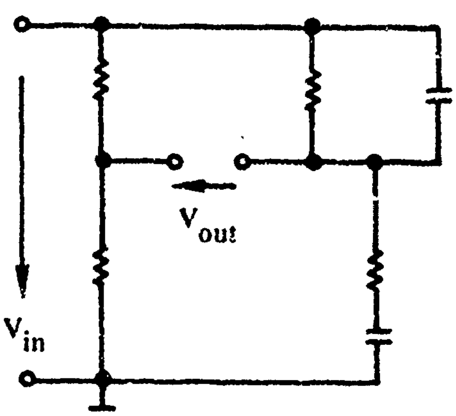

Figure 16. Basic all-pass network.

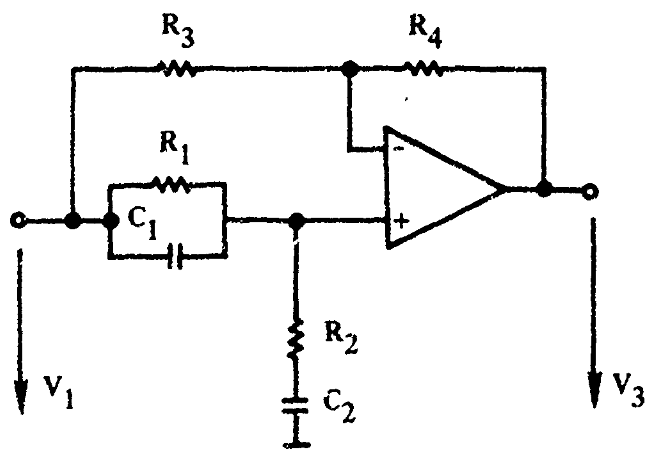

Figure 17. Sebol's all-pass circuit.

5.1 All-pass Condition

Comparing equation (39) with the general all-pass transfer function (31) shows that the constant terms of numerator and denominator are always identical, and, comparing with (16)

$$\omega_o = \frac{1}{\sqrt{C_1 C_2 R_1 R_2}} \, .$$

The other condition for an all-pass function is then

$$\frac{-1}{C_1 R_1} - \frac{1}{C_2 R_2} + \frac{k}{C_1 R_2} = \frac{1}{C_1 R_1} + \frac{1}{C_2 R_2} + \frac{1}{C_1 R_2}$$

or with the abbreviations

$$c = \frac{C_1}{C_2}$$

$$r = \frac{R_1}{R_2}$$

the all-pass condition is

$$k = \frac{2}{r} + 2c + 1. \qquad (40)$$

If it is met, the transfer function may be written as

$$\frac{V_3}{V_i} = \frac{\left(\frac{j\omega}{\omega_o}\right)^2 - \frac{j\omega}{\omega_o}\left(\frac{1}{\sqrt{rc}} + \sqrt{rc} + \sqrt{\frac{r}{c}}\right) + 1}{\left(\frac{j\omega}{\omega_o}\right)^2 + \frac{j\omega}{\omega_o}\left(\frac{1}{\sqrt{rc}} + \sqrt{rc} + \sqrt{\frac{r}{c}}\right) + 1} \, . \qquad (41)$$

5.2 Dimensioning

The desired filter parameter s provides again one relation between the circuit parameters c and r, so one of them is selectable. Comparing (41) with (31) and (16) shows that

$$s = \frac{1}{\sqrt{rc}} + \sqrt{rc} + \sqrt{\frac{r}{c}} \;;$$

this is solved for r to select :

$$\sqrt{r} = \frac{\sqrt{c}\, s}{2\,(c+1)} \pm \sqrt{\frac{c\, s^2}{4\,(c+1)^2} - \frac{1}{c+1}} \; . \tag{42}$$

Figure 18 shows the possible range of c, r, k in a typical case.

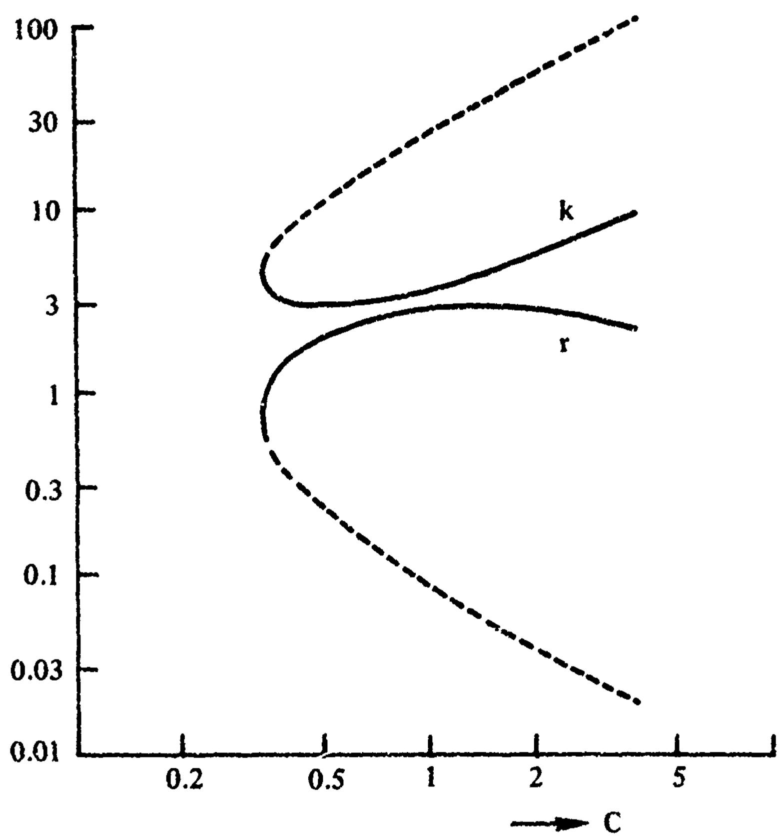

Figure 18. Solutions of r, k vs c for s = 4.
Solid curves are for the + sign in equation (42).

This all-pass realization contains only two capacitors, both of which can be selected. Furthermore, they may be the same. These are two attractive features of this circuit.

After C_1, C_2 and thus c have been selected, r and k are found from equations (42) and (40);

$$R_2 = \frac{1}{\omega_o \sqrt{C_1 C_2 r}} ,$$

and R_1 follows from r. The absolute values of R_3, R_4 do not enter; their ratio is given by k. The impedance level should be between the operational amplifier input and output impedances.

5.3 Effect of Component Tolerances

A worst-case analysis has been worked out similar to the one described in section 4.3. The computer program is listed in appendix 2. The analysis considers variations of the passive components only, not imperfections of the operational amplifier. As the latter is involved very tightly in the performance of the circuit, and finite gain, input impedance or common mode rejection certainly show an effect, the analysis is not as useful as in the other case. It shows necessary tolerances for the passive components; however, keeping these tolerances does not in itself guarantee the desired performance.

5.4 Example

Requirements and filter parameters as in section 4.5. For each channel, both capacitors are selected, and selected equal. For r, the + sign in equation (42) is used, because it results in a lower impedance spread.

Section 1	Section 2
$c = 1$	$c = 1$
$C_1 = C_2 = 10$ nF	$C_1 = C_2 = 2.2$ nF
$r = 2.646158$	$r = 2.646158$
$k = 3.755813$	$k = 3.755813$
$R_2 = 8.19$ kΩ	$R_2 = 8.85$ kΩ
R_1 21.7 kΩ	$R_1 = 23.4$ kΩ
R_3 5.6 kΩ	R_3 5.6 kΩ
R_4 21.0 kΩ	$R_4 = 21.0$ kΩ

The result of the worst-case analysis is shown in table II. R_1 and C_2 affect the phase difference mostly at lower frequencies, R_2 and C_1 mostly at higher frequencies. Deviations of R_3 and R_4 have a lesser effect at both ends of the band, none in the center. A tolerance of one percent of all passive components results in a worst-case error of almost 3.5 degrees in addition to the nominal error of two degrees. Resistor tolerances of 0.25 percent approximately halve the excess error.

The last result of table II shows that somewhat different circuit parameters give about the same tolerance sensitivity.

The sensitivity of this implementation is also about the same as the one treated in section 4.

6. SUMMARY

This report describes a simple method of determining the parameter and component values and tolerances for a class of 90 degree phase difference networks. After a short discussion of previously published papers on this subject and the limitations of their theoretical treatment, the transfer function on second order all-pass networks, the parameters of two such networks providing a 90-degree phase difference between their output signals, and the relationships of these parameters to the phase ripple and bandwidth are derived. Simple and straightforward computational methods, requiring only a desk calculator, for determining the filter parameters based on either the required bandwidth or the permissible phase deviation are then outlined.

Two different inductorless networks for the implementation of the all-pass function and their respective advantages and disadvantages are described, and equations defining component values are derived. Here, as with the filter parameters above, the flexibility and options available to the designer are stressed. For each circuit, a computer program is listed and discussed which permits the calculation of the worst effect of component tolerances on the phase difference. Transistor and integrated-circuit amplifiers suitable for the active parts of the circuits are considered briefly. Computational methods are illustrated by examples where the filter parameters, the circuit components, and worst-case phase errors for both implementations are calculated.

TABLE II. DOME CIRCUIT WITH DIFFERENTIAL AMPLIFIER

2. DEG. EQUAL-RIPPLE 90 DEG. DIFF. NW. WORST EFFECT OF COMP. VARIATIONS

```
NOMINAL BANDWIDTH F2/F1= 16.0800
NOM. DERIVED PARAM.   S=3.867941    B=2.050461    DELTA=2.00

NOM. REALIZ. CHAN.1   C= 1.00000    K= 3.75593    R= 2.64575
             CHAN.2      1.00000       3.75593       2.64575
*****************************************************************
                  F1     1.MAX    MIN    2.MAX    F2
                ***********************************
DP NOM./DEG      88.00   92.00   88.00   92.00   88.00

COMPONENT VARIATION (+/-PCT)  R1     R2     C1     C2     R3     R4
(OPPOSITE IN CHAN.1,2)       -----------------------------------------
                              1.00  -0.00  -0.00  -0.00  -0.00  -0.00

DP MAX /DEG      88.96   92.93   88.65   92.26   88.14
DP MIN /DEG      87.04   91.06   87.35   91.74   87.86

COMPONENT VARIATION (+/-PCT)  R1     R2     C1     C2     R3     R4
(OPPOSITE IN CHAN.1,2)       -----------------------------------------
                             -0.00   1.00  -0.00  -0.00  -0.00  -0.00

DP MAX /DEG      88.14   92.26   88.64   92.93   88.96
DP MIN /DEG      87.86   91.74   87.35   91.06   87.04

COMPONENT VARIATION (+/-PCT)  R1     R2     C1     C2     R3     R4
(OPPOSITE IN CHAN.1,2)       -----------------------------------------
                             -0.00  -0.00   1.00  -0.00  -0.00  -0.00

DP MAX /DEG      88.14   92.26   88.64   92.93   88.96
DP MIN /DEG      87.86   91.74   87.35   91.06   87.04

COMPONENT VARIATION (+/-PCT)  R1     R2     C1     C2     R3     R4
(OPPOSITE IN CHAN.1,2)       -----------------------------------------
                             -0.00  -0.00  -0.00   1.00  -0.00  -0.00

DP MAX /DEG      88.96   92.93   88.65   92.26   88.14
DP MIN /DEG      87.04   91.06   87.35   91.74   87.86

COMPONENT VARIATION (+/-PCT)  R1     R2     C1     C2     R3     R4
(OPPOSITE IN CHAN.1,2)       -----------------------------------------
                             -0.00  -0.00  -0.00  -0.00   1.00  -0.00

DP MAX /DEG      88.65   92.53   87.99   92.53   88.65
DP MIN /DEG      87.35   91.46   87.99   91.46   87.35

COMPONENT VARIATION (+/-PCT)  R1     R2     C1     C2     R3     R4
(OPPOSITE IN CHAN 1,2)       -----------------------------------------
                             -0.00  -0.00  -0.00  -0.00  -0.00   1.00

DP MAX /DEG      88.65   92.54   88.00   92.54   88.65
DP MIN /DEG      87.35   91.47   88.00   91.47   87.35

COMPONENT VARIATION (+/-PCT)  R1     R2     C1     C2     R3     R4
(OPPOSITE IN CHAN.1,2)       -----------------------------------------
                              1.00   1.00   1.00   1.00   1.00   1.00

DP MAX /DEG      91.45   95.43   90.59   95.43   91.45
DP MIN /DEG      84.45   88.51   85.39   88.51   84.46
```

TABLE II. DOME CIRCUIT WITH DIFFERENTIAL AMPLIFIER (Continued)

COMPONENT VARIATION (+/-PCT) (OPPOSITE IN CHAN.1,2)	R1	R2	C1	C2	R3	R4
	0.25	0.25	1.00	1.00	0.25	0.25

DP MAX /DEG	89.69	93.76	89.62	93.76	39.69
DP MIN /DEG	86.29	90.23	86.39	90.23	86.29

NOM. REALIZ.	CHAN.1	C= 0.50000	K= 3.15261	R= 1.73519
	CHAN.2	0.50000	3.15261	1.73519

****

	F1	1.MAX	MIN	2.MAX	F2
DP NOM./DEG	88.00	92.00	88.00	92.00	88.00

COMPONENT VARIATION (+/-PCT) (OPPOSITE IN CHAN.1,2)	R1	R2	C1	C2	R3	R4
	1.00	1.00	1.00	1.00	1.00	1.00

DP MAX /DEG	91.40	95.39	90.59	95.39	91.41
DP MIN /DEG	84.51	88.55	85.39	88.55	84.51

LITERATURE CITED

1. Dome, R.B., "Wideband Phase Shift Networks," Electronics, Vol 4, Dec 1946, p 112

2. Antony, R.T., "Dome Filter Design and Analysis Programs," HDL-TM-70-26, 1970

3. Sebol, R., "Design of Active 90-Degree Phase Difference Networks," HDL-TM-68-18

4. Bedrosian, S.D., "Normalized Design of 90-Degree Phase Difference Networks," IRE Trans. on Circuit Theory, June 1960

5. See, for instance, L. Weinberg, Network of Analysis and Synthesis, McGraw Hill, 1962, p 285

APPENDIX 1

COMPUTER PROGRAM FOR THE WORST-CASE ANALYSIS OF THE DOME FILTER CIRCUIT WITH BALANCED GENERATOR.

INPUT DATA :

Data	Format	Comment
f_2/f_1	F10.6	
c(chan. 1), c(chan. 2)	2F10.6	
ΔR_1, ΔR_2, ΔR_3, ΔC_1, ΔC_2, ΔC_3, ΔV	7F5.2	Relative deviations in percent (tolerance set)
. . . .		Any number of tolerance sets
blank		If analysis for different circuit parameters is desired *
c(chan. 1), c(chan. 2)		
. . . .		Any number of tolerance sets *

*As many sets of data cards may be used as desired.

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PROGRAM LISTING

```
      DIMENSION A(2),C(2),R(2),IA(7),D(8),Y(7,2),E(4,2),PEP(5),PEM(5),
     1PE(5)
      EQUIVALENCE (IA(1),I1),(IA(2),I2),(IA(3),I3),(IA(4),I4),
     1(IA(5),I5),(IA(6),I6),(IA(7),I7)
      PI=3.1415926
      READ(5,1) F
 1    FORMAT(2F10.6)
      X1=SQRT(1./F)
      X2=SQRT(F)
      G=((X1+X2)**2-4.)/(X1+X2-2.)
      S=SQRT(G+SQRT(G*G+2.*G*SQRT(2.*G-4.)))
      B=SQRT(1.+S*S/2.-G+SQRT((1.+S*S/2.-G)**2-1.))
      DELTA=2.*ATAN((S*S-2.*S*(B-1./B)-(B-1./B)**2)/(S*S+2.*S*(B-1./B)
     1-(B-1./B)**2))*180./PI
      WRITE(6,2)F,S,B,DELTA
 2    FORMAT(71H1      2.DEG.EQUAL-RIPPLE 90DEG.DIFF.NW. WORST EFFECT OF
     1COMP.VARIATIONS/25HONOMINAL BANDWIDTH F2/F1=,F8.4/25H NOM. DERIVED
     2 PARAM.    S=,F8.6,5H    B=,F8.6,9H    DELTA=,F4.2)
 3    READ(5,1) C
      DO 4  I=1,2
      R(I)=(-S/2./SQRT(C(I))+SQRT(S*S/4./C(I)+1.-1./C(I)))**2
 4    A(I)=0.5/(1./C(I)+R(I))-1.
      WRITE(6,5)(A(I),C(I),R(I),I=1,2)
 5    FORMAT(25HONOM. REALIZ. CHAN.1    A=,F8.5,5H    C=,F8.5,5H    R=,F8.5
     1/14X,6HCHAN.2,3F13.5/1X,58(1H*))
      DO 6 N=1,8
 6    D(N)=0.
      GO TO 12
 7    READ(5,8)(D(N),N=1,7)
 8    FORMAT(7F5.2)
      IF(D(1).EQ.0..AND.D(2).EQ.0..AND.D(3).EQ.0..AND.D(4).EQ.0..AND.
     1D(5).EQ.0..AND.D(6).EQ.0..AND.D(7).EQ.0.) GO TO 3
      DO 10  I=1,5
      PEP(I)=0.
 10   PEM(I)=PI
 12   DO 13  N=1,7
      IA(N)=1
 13   IF(D(N).NE.0.) IA(N)=2
      DO 50  N1=1,I1
      Y(1,1)=1.+D(1)/100.*(-1.)**N1
      Y(1,2)=1.-D(1)/100.*(-1.)**N1
      DO 50  N2=1,I2
      Y(2,1)=1.+D(2)/100.*(-1.)**N2
      Y(2,2)=1.-D(2)/100.*(-1.)**N2
      DO 50  N3=1,I3
      Y(3,1)=1.+D(3)/100.*(-1.)**N3
      Y(3,2)=1.-D(3)/100.*(-1.)**N3
      DO 50  N4=1,I4
      Y(4,1)=1.+D(4)/100.*(-1.)**N4
      Y(4,2)=1.-D(4)/100.*(-1.)**N4
      DO 50  N5=1,I5
      Y(5,1)=1.+D(5)/100.*(-1.)**N5
      Y(5,2)=1.-D(5)/100.*(-1.)**N5
      DO 50  N6=1,I6
      Y(6,1)=1.+D(6)/100.*(-1.)**N6
```

```
      Y(6,2)=1.-D(6)/100.*(-1.)**N6
      DO 50  N7=1,I7
      Y(7,1)=1.+D(7)/100.*(-1.)**N7
      Y(7,2)=1.-D(7)/100.*(-1.)**N7
      DO 15  I=1,2
      E(1,I)=SQRT(C(I)/R(I))/Y(1,I)/Y(5,I)/Y(7,I)-1./SQRT(R(I)*C(I))
     1/Y(1,I)/Y(4,I)-SQRT(R(I)*C(I))/Y(2,I)/Y(5,I)
      E(2,I)=1./Y(1,I)/Y(2,I)/Y(4,I)/Y(5,I)
      E(3,I)=SQRT(R(I)*C(I))*(1./Y(2,I)+1./A(I)/Y(3,I))/(Y(5,I)+Y(6,I)
     1/A(I))+1./SQRT(R(I)*C(I))/Y(1,I)/Y(4,I)+SQRT(C(I)/R(I))/Y(1,I)/
     2 (Y(5,I)*Y(6,I)/A(I))
15    E(4,I)=(1./Y(2,I)+1./Y(3,I)/A(I))/Y(1,I)/Y(4,I)/(Y(5,I)+Y(6,I)/
     1A(I))
      N=1
      DO 30  I=1,101
      IF(N.EQ.4.AND.I.LT.101) GO TO 30
      X=X1*(X2/X1)**(FLOAT(I-1)/100.)
      P=-ARCT(-X*B*E(1,1),E(2,1)-(X*B)**2)+ARCT(X*B*E(3,1),E(4,1)-(X*B)
     1**2)+ARCT(-X/B*E(1,2),E(2,2)-(X/B)**2)-ARCT(X/B*E(3,2),E(4,2)-
     2(X/B)**2)
      IF(I-1) 20,20,21
20    PE(1)=P
      GO TO 30
21    IF(P-P0) 26,30,22
22    IF(N-2) 23,24,25
23    PE(2)=P
      GO TO 30
24    N=3
      GO TO 30
25    PE(4)=P
      GO TO 30
26    IF(N-2) 27,28,29
27    N=2
      GO TO 30
28    PE(3)=P
      GO TO 30
29    N=4
30    P0=P
      PE(5)=P
      DO 35  I=1,5
      IF(PE(I).GT.PEP(I)) PEP(I)=PE(I)
35    IF(PE(I).LT.PEM(I)) PEM(I)=PE(I)
50    CONTINUE
      IF(D(8)) 51,51,60
51    DO 52  I=1,5
52    PE(I)=PE(I)/PI*180.
      WRITE(6,55) PE
55    FORMAT(17X,2HF1,5X,5H1.MAX,4X,3HMIN,4X,5H2.MAX,4X,2HF2/
     115X,38(1H*)/13H DP NOM./DEG ,5F8.2)
      D(8)=1.
      GO TO 7
60    DO 62  I=1,5
      PEP(I)=PEP(I)/PI*180.
62    PEM(I)=PEM(I)/PI*180.
      WRITE(6,65) (D(N),N=1,7),PEP,PEM
65    FORMAT(69H0COMPONENT VARIATION (+/-PCT)  R1     R2     R3     C1     C
     12     C3     DV/23H (OPPOSITE IN CHAN.1,2),6X,42(1H-)/28X,7F6.2
     3/13H0DP MAX /DEG ,5F8.2/13H DP MIN /DEG ,5F8.2)
       GO TO 7
       END
```

APPENDIX 2

COMPUTER PROGRAM FOR THE WORST-CASE ANALYSIS OF THE DOME FILTER CIRCUIT WITH OPERATIONAL AMPLIFIER

INPUT DATA :

Data	Format	Comment
f_2/f_1	F10.6	
c(chan. 1), c(chan. 2)	2F10.6	
ΔR_1, ΔR_2, ΔC_1, ΔC_2, ΔR_3, ΔR_4	6F5.2	Tolerance in percent (tolerance set)
. . . .		Any number of tolerance sets
blank		If analysis for different circuit parameters is desired *
c(chan. 1), c(chan. 2)		
. . . .		Any number of tolerance sets *

*As many sets of data cards may be used as desired.

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PROGRAM LISTING

```
C DOME FILTER (DIFF.AMPL.), WORST CASE
      DIMENSION C(2),R(2),IA(6),D(7),Y(6,2),E(3,2),
     1PEP(5),PEM(5),PE(5)
      EQUIVALENCE (IA(1),I1),(IA(2),I2),(IA(3),I3),(IA(4),I4),
     1(IA(5),I5),(IA(6),I6)
      REAL K(2)
      PI=3.1415926
      READ(5,1) F
    1 FORMAT(2F10.6)
      X1=SQRT(1./F)
      X2=SQRT(F)
      G=((X1+X2)**2-4.)/(X1+X2-2.)
      S=SQRT(G+SQRT(G*G+2.*G*SQRT(2.*G-4.)))
      B=SQRT(1.+S*S/2.-G+SQRT((1.+S*S/2.-G)**2-1.))
      DELTA=2.*ATAN((S*S-2.*S*(B-1./B)-(B-1./B)**2)/(S*S+2.*S*(B-1./B)
     1-(B-1./B)**2))*180./PI
      WRITE(6,2)F,S,B,DELTA
    2 FORMAT(71H1      2.DEG.EQUAL-RIPPLE 90DEG.DIFF.NW. WORST EFFECT OF
     1COMP.VARIATIONS/25HQNOMINAL BANDWIDTH F2/F1=,F8.4/25H NOM. DERIVED
     2 PARAM.    S=,F8.6,5H    B=,F8.6,9H    DELTA=,F4.2)
    3 READ(5,1) C
      DO 4  I=1,2
      G=SQRT(C(I))*S/2./(1.+C(I))
      R(I)=(G+SQRT(G*G-1./(1.+C(I))))**2
    4 K(I)=2./R(I)+2.*C(I)+1.
      WRITE(6,5)(C(I),K(I),R(I),I=1,2)
    5 FORMAT(25HONOM. REALIZ. CHAN.1   C=,F8.5,5H    K=,F8.5,5H    R=,F8.5
     1/14X,6HCHAN.2,3F13.5/1X,58(1H*))
      DO 6  N=1,7
    6 D(N)=0.
      GO TO 12
    7 READ(5,8)(D(N),N=1,6)
    8 FORMAT(7F5.2)
      IF(D(1).EQ.0..AND.D(2).EQ.0..AND.D(3).EQ.0..AND.D(4).EQ.0..AND.
     1D(5).EQ.0..AND.D(6).EQ.0.) GO TO 3
      DO 10  I=1,5
      PEP(I)=0.
   10 PEM(I)=PI
   12 DO 13  N=1,6
      IA(N)=1
   13 IF(D(N).NE.0.) IA(N)=2
      DO 50  N1=1,I1
      Y(1,1)=1.+D(1)/100.*(-1.)**N1
      Y(1,2)=1.-D(1)/100.*(-1.)**N1
      DO 50  N2=1,I2
      Y(2,1)=1.+D(2)/100.*(-1.)**N2
      Y(2,2)=1.-D(2)/100.*(-1.)**N2
      DO 50  N3=1,I3
      Y(3,1)=1.+D(3)/100.*(-1.)**N3
      Y(3,2)=1.-D(3)/100.*(-1.)**N3
      DO 50  N4=1,I4
      Y(4,1)=1.+D(4)/100.*(-1.)**N4
      Y(4,2)=1.-D(4)/100.*(-1.)**N4
      DO 50  N5=1,I5
      Y(5,1)=1.+D(5)/100.*(-1.)**N5
```

```
      Y(5,2)=1.-D(5)/100.*(-1.)**N5
      DO 50  N6=1,I6
      Y(6,1)=1.+D(6)/100.*(-1.)**N6
      Y(6,2)=1.-D(6)/100.*(-1.)**N6
      DO 15  I=1,2
      G=1./Y(1,I)/Y(3,I)/SQRT(C(I)*R(I))
      H=SQRT(C(I)*R(I))/Y(2,I)/Y(4,I)
      E(1,I)=G+H-SQRT(R(I)/C(I))*K(I)/Y(2,I)/Y(3,I)/Y(5,I)*Y(6,I)
      E(2,I)=1./Y(1,I)/Y(2,I)/Y(3,I)/Y(4,I)
   15 E(3,I)=G+H+SQRT(R(I)/C(I))/Y(2,I)/Y(3,I)
      N=1
      DO 30  I=1,101
      IF(N.EQ.4.AND.I.LT.101) GO TO 30
      X=X1*(X2/X1)**(FLOAT(I-1)/100.)
      G=E(2,1)-(X*B)**2
      H=E(2,2)-(X/B)**2
      P=-ARCT(X*B*E(1,1),G)+ARCT(X*B*E(3,1),G)+ARCT(X/B*E(1,2),H)-
     1ARCT(X/B*E(3,2),H)
      IF(I-1) 20,20,21
   20 PE(1)=P
      GO TO 30
   21 IF(P-PO) 26,30,22
   22 IF(N-2) 23,24,25
   23 PE(2)=P
      GO TO 30
   24 N=3
      GO TO 30
   25 PE(4)=P
      GO TO 30
   26 IF(N-2) 27,28,29
   27 N=2
      GO TO 30
   28 PE(3)=P
      GO TO 30
   29 N=4
   30 PO=P
      PE(5)=P
      DO 35  I=1,5
      IF(PE(I).GT.PEP(I)) PEP(I)=PE(I)
   35 IF(PE(I).LT.PEM(I)) PEM(I)=PE(I)
   50 CONTINUE
      IF(D(7)) 51,51,60
   51 DO 52  I=1,5
   52 PE(I)=PE(I)/PI*180.
      WRITE(6,55) PE
   55 FORMAT(17X,2HF1,5X,5H1.MAX,4X,3HMIN,4X,5H2.MAX,4X,2HF2/
     115X,38(1H=)/13H DP NOM./DEG ,5F8.2)
      D(7)=1.
      GO TO 7
   60 DO 62  I=1,5
      PEP(I)=PEP(I)/PI*180.
   62 PEM(I)=PEM(I)/PI*180.
      WRITE(6,65)(D(N),N=1,6),PEP,PEM
   65 FORMAT(63HOCOMPONENT VARIATION (+/-PCT)  R1     R2     C1     C2     R
     13     R4/23H (OPPOSITE IN CHAN.1,2),6X,35(1H-)/28X,6F6.2/
     213HODP MAX /DEG ,5F8.2/13H DP MIN /DEG ,5F8.2)
      GO TO 7
      END
```

APPENDIX 3

SUBPROGRAM ARCT REQUIRED BY PROGRAMS IN APPENDIXES 1 AND 2

```
C QUADRANT-CORRECT (+/-180 DEG) ARCTG(NUM.,DENOM.) SUBPROGRAM
      FUNCTION ARCT(XI,XR)
      PI=3.1415926
      IF(XR) 91,94,90
90    ARCT=ATAN(XI/XR)
      RETURN
91    IF(XI) 93,92,92
92    ARCT=ATAN(XI/XR) +PI
      RETURN
93    ARCT=ATAN(XI/XR)-PI
      RETURN
94    IF(XI) 96,97,95
95    ARCT=PI/2.
      RETURN
96    ARCT=-PI/2.
      RETURN
97    ARCT=0.
      RETURN
      END
```